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A plastic flow-induced fracture theory for fatigue crack growth

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A plastic flow-induced fracture theory for fatigue crack growth is presented. A new formulae for the fatigue stress intensity threshold and the fatigue crack growth rate law are derived by applying the principle of energy conservation in considering the fatigue crack growth process in the presence of local plastic flow ahead of the crack-tip. The present theory predicts not only the fatigue crack growth rate being just proportional to the rate of creation of dislocation at the crack-tip, but also the fatigue stress intensity threshold, which can be determined according to the applied fatigue stress amplitude and the characteristic size of microstructural fracture process ahead of the crack-tip, and can account for the fatigue crack growth characteristics at both low and high levels of applied fatigue stress intensity amplitude. All the results are universal and agree with the existing empirical results and experimental observations.

1. Introduction

It has been recognized that the behaviour of material near the crack-tip will ultimately form the basis for all fracture theories. In order to build up a fatigue fracture theory which describes the cyclic plastic deformation based on the behaviour of dislocations near the crack-tip, much work has been done [1–13]; however, dislocation-enhanced fatigue crack growth (FCG) has not been analysed by applying rigorous energetic considerations, and it is the purpose of the present paper to present a simple model that allows such a calculation to be made of the effects of the plastic flow on the growth dynamics of a fatigue crack by applying the principle of energy conservation. The main conclusion derived from the present study includes a new expression for threshold stress intensity range and a new formula for the FCG rate.

2. The present theory

The two-dimensional plane strain model shown in Fig. 1 is proposed to explain plastic flow-enhanced FCG. Here we use a giant or super dislocation, nb, to simulate the localized cyclic plasticity around the fatigue crack-tip. The physical meanings can be explained as n dislocations generated by one cycle stress at $x = d^*$ ahead of the crack-tip, interacting with the crack which results in growth of the crack. In our consideration, all of the microplasticity seems to be concentrated in the slip-bands, reversed dislocation motion precedes local fracture and lowers the cohesive energy of the slip planes, effectively lowering local stress σ_f, until local cleavage fracture occurs; or in other words, only if the localized plastic flow reaches a critical state does the secondary crack initiate in front of the crack-tip and the main crack propagate [6]. The characteristic length, d^*, reflects the sensitive degree of materials microproperties to applied loads as well as test environments and cyclic frequencies, and is a very important geometric parameter which can be determined by the microstructure of the materials. It is a typical fracture in which the secondary cracks nucleate, grow and coalesce with the main crack due to dislocation pile-ups caused by local plastic flow against the microstructure defects within the region ahead of the crack-tip [6]. In this paper the FCG dynamics is discussed quantitatively.

According to the well known theory of fracture mechanics, when the applied stress intensity range, $\Delta K \equiv K_{max} - K_{min}$, exceeds the threshold value, ΔK_{th}, i.e. $\Delta K > \Delta K_{th}$, then the fatigue crack grows in a steady stage. However, in physics, the total energy of the crack system sketched in Fig. 1 is always kept at conservation. With this consideration we generally have [14]

$$\frac{d}{dN}(T + U) = 0 \tag{1}$$

where T and U are the kinetic energy and static energy of the fatigue crack system, respectively.

According to dislocation theory [15], the static energy for the fatigue crack system sketched in Fig. 1 is

$$U = 4G_{IC}a - \frac{1}{2}\Delta\sigma b \int_{-a}^{+a} dx \int_{x}^{+a} \Delta f(x')dx' \tag{2}$$

where the first term represents effective surface energy of the crack of size $2a$ and the second term is the

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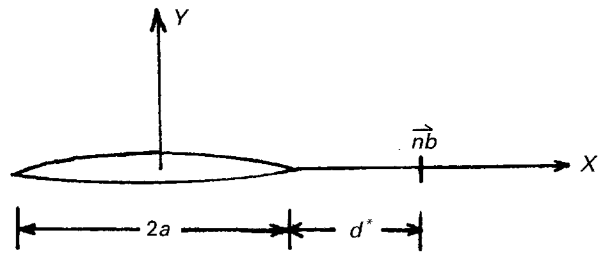

Figure 1 The dislocation model of fatigue crack growth in the presence of local plastic flow ahead of the crack-tip.

mechanical energy of the crack dislocation systems. Here G_{IC} is the fatigue crack extension force, $\Delta\sigma$ is the applied cyclic stress amplitude and $\Delta f(x)$ is the dislocation densities of the crack body which can be calculated from

$$\int_{-a}^{+a} \frac{A\Delta f(x')\mathrm{d}x'}{x-x'} + \frac{An}{a+d^*-x} + \Delta\sigma = 0 \qquad (3)$$

For $|x| \leqslant a$, the Cauchy principal value of the singular integral is understood; where $A = \mu b/2\pi(1-\nu)$; μ is the shear modulus and ν is Poisson's ratio.

It should be pointed out here that, in using this approach, we have made two assumptions: (1) by using the superdislocation approach, we have eliminated dislocation–dislocation interactions, a simplifying approximation; (2) no friction force has been included as a back-stress on dislocation motion which can have a substantial effect in strain rate sensitive materials at high loading frequencies. With these two simplifying assumptions, the exact solution of the singular integral in Equation 3 is [16]

$$\Delta f(x) = \frac{-1}{\pi^2 A(a^2-x^2)^{1/2}} \int_{-a}^{+a} \frac{(a^2-x'^2)^{1/2}}{x'-x} \times \left(\frac{An}{a+d^*-x'} + \Delta\sigma\right)\mathrm{d}x' + \frac{K}{(a^2-x^2)^{1/2}} \qquad (4)$$

where K is a constant that has to be determined from the physical condition that

$$\int_{-a}^{+a} \Delta f(x)\,\mathrm{d}x = 0 \qquad (5)$$

When K is calculated and its value substituted into Equation 4, the dislocation distribution function $\Delta f(x)$ becomes

$$\Delta f(x) = \frac{\Delta\sigma}{\pi A}\frac{x}{(a^2-x^2)^{1/2}} - \frac{n}{\pi(a^2-x^2)^{1/2}} + \frac{n((a+d^*)^2-a^2)^{1/2}}{\pi(a^2-x^2)^{1/2}(a+d^*-x)} \qquad (6)$$

Inserting Equation 6 into Equation 2 gives

$$U = 4G_{IC}a - \frac{\pi(1-\nu^2)}{E}\Delta\sigma^2 a^2 - \Delta\sigma nb \times \left(a + \frac{d^*}{2} - \frac{1}{4}[d^*(2a+d^*)]^{1/2}\right) \qquad (7)$$

It may be proven easily from Equation 7 that the fatigue crack begins to propagate with rapid speed when

$$\left\{\begin{array}{l} nb = (nb)_{\text{critical}} \simeq \dfrac{2\pi(1-\nu^2)\Delta\sigma a}{E} \\ a = a_c = \dfrac{EG_{IC}}{\pi(1-\nu^2)\Delta\sigma^2} \end{array}\right\} \qquad (8)$$

It is also not difficult to find the kinetic energy of a fatigue crack as [17]

$$T = \frac{k\rho_0 a^2 \dot{a}^2 \omega^2 \Delta\sigma^{4/(m+1)}}{2E^2\sigma_A^{2/(m+1)}} \qquad (9)$$

where E is Young's modulus, ρ_0 is metal density, σ_A is strength coefficient of metal, m is hardening exponent, $k \simeq 5.45$, ω is the cyclic frequency and $\dot{a} = \mathrm{d}a/\mathrm{d}N$ is the rate of FCG.

Inserting Equations 7 and 9 into Equation 1, we obtain the dynamic equation of plastic flow enhanced FCG under the action of constant cyclic stress amplitude as follows

$$a^2\dot{a}\ddot{a} + a\dot{a}^3 + \frac{E^2\sigma_A^{2/(m+1)}}{k\rho_0\omega^2\Delta\sigma^{4/(m+1)}}\left\{4G_{IC} - \frac{2\pi(1-\nu^2)}{E}\Delta\sigma^2 a - \Delta\sigma nb \times \left[1 - \frac{1}{4}\left(\frac{d^*}{2a+d^*}\right)^{1/2}\right]\right\}\dot{a} - \frac{E^2\sigma_A^{2/(m+1)}}{k\rho_0\omega^2\Delta\sigma^{(3-m)/(1+m)}} \times \left\{a + \frac{d^*}{2} - \frac{1}{4}[d^*(2a+d^*)]^{1/2}\right\}\frac{\mathrm{d}(nb)}{\mathrm{d}N} = 0 \qquad (10)$$

where $\ddot{a} = \mathrm{d}\dot{a}/\mathrm{d}N$ is the growth rate of acceleration for a fatigue crack; $\mathrm{d}(nb)/\mathrm{d}N$ is the number of dislocations generated per cycle from the crack-tip. In obtaining Equation 10, we have used the condition that rate of change of cyclic stress amplitude per cycle is $\mathrm{d}\Delta\sigma/\mathrm{d}N = 0$.

Since Equation 10 is a non-linear equation, generally speaking, it is very difficult to get a simple expression of $\dot{a}$ from it. But, for small $\dot{a}$ and $a \ll a_c$, i.e. for slow growth stage of crack, $\ddot{a}$ and $\dot{a}^3$ may be negligible, then substituting Equation 8 into Equation 10 gives

$$\frac{\mathrm{d}a}{\mathrm{d}N} = \frac{\Delta\sigma\{a + d^*/2 - 1/4[d^*(2a+d^*)]^{1/2}\}}{4G_{IC} - [2\pi(1-\nu^2)/E]\,\Delta\sigma^2 a - \Delta\sigma nb\{1 - 1/4[d^*/(2a+d^*)]^{1/2}\}}\frac{\mathrm{d}(nb)}{\mathrm{d}N} \qquad (11)$$

This is a rate expression of crack-tip cyclic plastic flow enhanced FCG. Of special interest in this expression is that it predicts the FCG rate being just proportional to the rate of creation of dislocation at the crack-tip. It is also evident that at this stage that the growth rate of the fatigue crack is closely related to $\mathrm{d}(nb)/\mathrm{d}N > 0$, i.e.

to the number of dislocations generated at the crack-tip. If $d(nb)/dN = 0$, then $da/dN = 0$, and no FCG occurs.

3. Results and discussion

In order to further find a formulation for da/dN in Equation 11, the following approximation is used to calculate $d(nb)/dN$ and (nb). Let n_0 be the average number of mobile dislocation sources per unit volume around the crack-tip, and L the average length of slip plane. The plastic strain in one cycle can then be expressed as [14]

$$n_0 L^2 \frac{d(nb)}{dN} = \Delta\varepsilon_p \tag{12}$$

For metal fatigue there is a universal empirical formula [18]

$$\Delta\varepsilon_p = \varepsilon_f \left(\frac{\Delta\sigma}{\sigma_f}\right)^{1/\beta} \tag{13}$$

where β is the cyclic strain hardening exponent and $0.10 < \beta < 0.20$ for all metallurgical materials [18]; both σ_f and ε_f are the real fracture stress and the real fracture strain in monotonic loading, respectively. From Equations 12 and 13 we have

$$\frac{d(nb)}{dN} = \frac{\varepsilon_f}{n_0 L^2} \left(\frac{\Delta\sigma}{\sigma_f}\right)^{1/\beta} \tag{14}$$

With the fatigue crack approach to propagation, Relation 8 between (nb) and a is approximate. Furthermore, with a view to reveal the nature of FCG process and form a concise expression of FCG rate, we consider here a simple mathematical transformation

$$a + \frac{d^*}{2} - \frac{1}{4}(d^*(2a + d^*))^{1/2} \simeq (a^{1/2} - Yd^{*1/2})^2 \tag{15}$$

here Y is a numerical constant factor and numerically

$$Y \simeq 0.18 \sim 0.70 \tag{16}$$

The former ($Y = 0.18$) corresponds to $d^* \ll a$, and the latter ($Y = 0.70$) corresponds to $d^* \sim (2a)/3$. Clearly, Y has obvious physical meanings which also reflect the sensitive degree of material's microproperties to applied cyclic loads as well as test environments and cyclic frequencies.

Substituting Equations 8, 14 and 15 into Equation 11 gives

$$\frac{da}{dN} = \frac{E\varepsilon_f \Delta\sigma^{[(1/\beta)-1]}}{2\pi(1 - \nu^2) n_0 L^2 \sigma_f^{1/\beta}} \times \frac{(\Delta K - \Delta K_{th})^2}{(K_{IC}^2 - \Delta K^2)} \tag{17}$$

where we have defined

$$\Delta K = \Delta\sigma\,(\pi a)^{1/2} \tag{18a}$$

and

$$\Delta K_{th} = Y\Delta\sigma(\pi d^*)^{1/2} \tag{18b}$$

as crack-tip stress intensity range and threshold stress intensity range, respectively.

Equations 17 and 18b are the new rate formula of plastic flow enhanced FCG and the new expression for threshold stress intensity range derived by applying the principle of energy conservation in considering the FCG process.

Of most importance and interest in Equation 17 is that it can account for the crack growth characteristics at both low and high levels of ΔK. For low values of ΔK, i.e. $K_{IC} \gg \Delta K$, Equation 17 can be approximately written as

$$\frac{da}{dN} = \frac{E\varepsilon_f \Delta\sigma^{[(1/\beta)-1]}}{2\pi(1 - \nu^2) n_0 L^2 \sigma_f^{1/\beta} K_{IC}^2}(\Delta K - \Delta K_{th})^2 \tag{17a}$$

Such a rate expression of FCG agrees completely with the empirical relation suggested by Donahue *et al.* [19] and agrees with the Paris empirical formula, if only from the point of view of $da/dN \sim (\Delta K)^2$. So, our new expression of FCG rate law (Equation 17) can be used to account for the experimental data at low stress levels and the existence of a threshold value ΔK_{th} of ΔK at which no crack propagation occurs, since if $\Delta K \to \Delta K_{th}$, then $da/dN \to 0$.

At high ΔK values, i.e. $\Delta K \gg \Delta K_{th}$, we can write Equation 17 as

$$\frac{da}{dN} = \frac{E\varepsilon_f \Delta\sigma^{[(1/\beta)-1]}}{2\pi(1 - \nu^2) n_0 L^2 \sigma_f^{1/\beta}} \frac{\Delta K^2}{(K_{IC}^2 - \Delta K^2)} \tag{17b}$$

It is evident that Equation 17b agrees with Forman *et al.* [20] empirical formula only from the point of view of $da/dN \sim \Delta K^2/(K_{IC}^2 - \Delta K^2)$. It should be pointed out, however, although at first sight it seems that $da/dN \to \infty$ as $\Delta K \to K_{IC}$, from Equation 17, this is not so, because the result calculated from Equation 10 told us that it is only suitable for a small growth rate and not for a high propagation rate like $\Delta K \to K_{IC}$. As a matter of fact, as $\Delta K \to K_{IC}$, or $a \to a_c$, the crack length approaches critical size, a_c and its growth rate will increase abruptly. Substituting Equation 8 into Equation 10, and ignoring $\ddot{a}$, we get the growth rate at the critical point (for $m = 1$)

$$a \to a_c, \dot{a} \to \dot{a}_{max} = \left[\frac{\pi(1 - \nu^2)E}{2k\omega^2\rho_0}\right]^{1/2} \tag{19}$$

which is close to the speed of the stress wave propagating in metal $(E/\rho_0)^{1/2}$, and at this time fracture occurs. This is, in general, the so-called true physical meaning of $\dot{a} \to \infty$.

It should be emphasized that Equation 18b is fundamentally interesting. This is a new definition of threshold stress intensity range that has not been found in the literature. The concept of the fatigue stress intensity threshold has developed over the last 20 years into a useful parameter for the characterization of materials. However, there are still many difficulties involved in the determination of this parameter. Here we emphasise that, according to Equation 18b, ΔK_{th} can be dealt with as a material parameter. As a matter of fact, ΔK_{th} can generally be determined by rigorous calculation as follows.

To calculate the crack-tip stress intensity range in the existence of local cyclic plastic flow at a microscopic level, we have the formula [21]

$$\Delta K^{tip} = \pi(2\pi)^{1/2} A \lim_{x \to a}(a - x)^{1/2}\,\Delta f(x) \tag{20}$$

where A and $\Delta f(x)$ are as before. Inserting Equation 6 into Equation 20 it is very easy to get

$$\Delta K^{\mathrm{tip}} = \Delta\sigma(\pi a)^{1/2} + \frac{\mu n b}{2(1-\nu)(\pi a)^{1/2}}\left[\left(1+\frac{2a}{d^*}\right)^{1/2} - 1\right] \quad (21)$$

Equation 21 is the crack-tip stress intensity range. It is clear that ΔK^{tip} consists of two parts: the applied stress intensity (the first term on the right side of Equation 21) and the contribution of local cyclic plastic flow (the second term on the right side of Equation 21).

When $n = 0$, Equation 21 reduces to the well known definition of applied stress intensity range. From this it can be seen that it includes all the information of applied loading and microstructure conditions. In fracture mechanics once $\Delta K^{\mathrm{tip}} = K_{\mathrm{IC}}$, then the crack begins to propagate. Accordingly

$$\Delta K_{\mathrm{th}} = K_{\mathrm{IC}} - \frac{E(nb)_{\mathrm{critical}}}{4\pi(1-\nu^2)(\pi a)^{1/2}}\left[\left(1+\frac{2a}{d^*}\right)^{1/2} - 1\right] \quad (22)$$

here $(nb)_{\mathrm{critical}}$ physically means critical cyclic plastic flow for the secondary microcrack nucleation and propagation ahead of the crack-tip. Remembering Equation 10 and taking the approximate calculation $[1 + (2a/d^*)]^{1/2} - 1 \simeq (2a/d^*)^{1/2}$, it is easy, from Equation 22, to find

$$\Delta K_{\mathrm{th}} = \frac{K_{\mathrm{IC}}}{2}\left\{1 - \left[1 - \frac{2(2)^{1/2}}{\pi} Y\right]^{1/2}\right\} \quad (23)$$

In obtaining Equation 23 we have used the definition

$$\Delta K_{\mathrm{th}} = Y\Delta\sigma(\pi d^*)^{1/2}$$

i.e. Equation 18b. Clearly, ΔK_{th} can be dealt with a material constant for any given Y and K_{IC}, and, numerically

$$\Delta K_{\mathrm{th}} = 0.085 K_{\mathrm{IC}} \sim 0.392 K_{\mathrm{IC}} \quad (24)$$

which just corresponds to

$$Y = 0.18 \sim 0.70$$

i.e. Equation 16.

To summarize, it is evident from the above discussion that the characteristic size, d^*, plays a critical role in determining the threshold stress range and the FCG rate. In the present approach we have considered d^* to be a material parameter under one given loading condition, and it can be further understood as the characteristic size of the microstructural fracture process [22]. The location for $x = d^*$ is always at a site where the local tensile stress amplitude is at a maximum. It is thought that the macro incontinuity at sites near the location of the local tensile stress maximum solely nucleates the microcrack, and the more cyclic plastic flow accumulation results in secondary crack initiation. With the propagation of the secondary crack, and coalescence with the main crack, the main crack advance finally occurs. Experimental evidence support us in making such a consideration – the basic fact is the occurrence of microscopic damage ahead of the growing crack [6]. When the stress intensity range is large, the plastic zone at the crack-tip will also be large, and it is to be expected that microcracks will form ahead of the main crack. These will occur at grain boundaries, or other obstacles slip, particularly at brittle precipitates, just as in the case of fracture under monotonic loading.

In order to have a quantitative knowledge of the present discussion, the FCG rate is calculated from Equation 17 for mild steel as shown in Fig. 2. For the calculation, respective parameters are assumed as follows: E, $2\times10^4\,\mathrm{kg\,mm^{-2}}$; β, 0.13; ν, 0.25; n_0, $5\times10^6\,\mathrm{mm^{-3}}$; L, 10^{-2} mm; σ_f, $80\,\mathrm{kg\,mm^{-2}}$; ε_f, 1.04; $\Delta\sigma$, $20\,\mathrm{kg\,mm^{-2}}$, K_{IC}, $2.50\times10^2\,\mathrm{kg\,mm^{-3/2}}$; and also $\Delta K_{\mathrm{th}} = 0.1 K_{\mathrm{IC}}$ follows from Equation 23.

It is obvious from Fig. 2 that the theoretical curve of the FCG rate agrees completely with the typical form of the FCG rate curve that has been found experimentally. Similarly, three regions, according to the curve shape in Fig. 2 can be distinguished. In region I, $\mathrm{d}a/\mathrm{d}N$ diminishes rapidly to a vanishingly small level and there is a threshold value of the stress intensity range amplitude ΔK_{th} meaning that for $\Delta K < \Delta K_{\mathrm{th}}$ no crack growth takes place. In region II there is a deterministic semi-logarithmic $\Delta K - \ln(\mathrm{d}a/\mathrm{d}N)$ relationship. Finally, in region III the FCG rate curve rises and the maximum stress intensity, $K_{\max}$, in the fatigue load cycle becomes equal to the critical stress

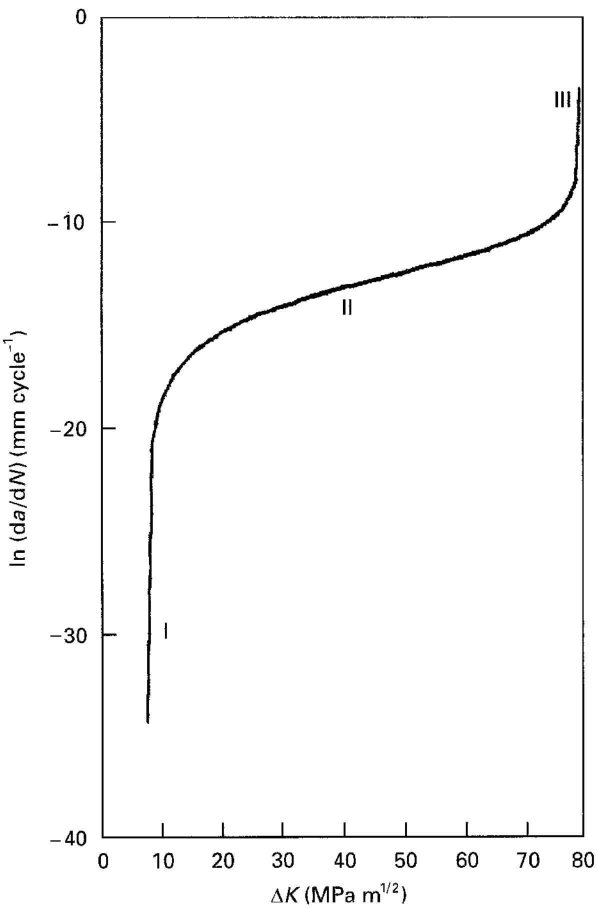

Figure 2 The theoretical curve of fatigue crack growth rate for mild steel according to Equation 17 ($1\ \mathrm{kg\,mm^{-3/2}} = 0.3162\ \mathrm{MPa\,m^{1/2}}$; $1\ \mathrm{MPa\,m^{1/2}} = 0.91\ \mathrm{ksi}\sqrt{\mathrm{in}}$).

intensity, K_{IC}, leading to catastrophic failure. All the results are well known experimental facts for FCG [23].

4. Conclusions

In conclusion, a new theory has been proposed and developed for fatigue fracture based on FCG due to plastic flow on a localized scale within the crack-tip plastic zone. A new formulae for threshold stress intensity range and FCG rate law are derived by applying the principle of energy conservation in considering the FCG process in the presence of local plastic flow ahead of the crack-tip. The present theory predicts not only the fatigue crack growth rate being proportional to the rate of creation of dislocation at the crack-tip, but also the fatigue stress intensity threshold which can be determined according to the applied fatigue stress amplitude and the characteristic size of microstructural fracture process ahead of the crack-tip; and can account for the fatigue crack growth characteristics at both low and high levels of applied fatigue stress intensity amplitude. All the results are universal and agree with the existing empirical results and experimental facts. Although the new formulae for both fatigue stress intensity threshold and FCG rate law can explain experimental observations, additional theoretical studies are suggested to confirm the proposed fatigue fracture theory since it is of great theoretical and technological importance.

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